Learning Curve Theory

LEARNING OBJECTIVES:
After studying this unit, you will be able to:
- Understanding of learning curve phenomenon.
- Understand how the percentage learning rate applies to the doubling of output.
- Know that cumulative average time means the average time per unit for all units produced so far back to and including the first unit produced.
- Use of learning curve theory in such situations as pricing decisions, work scheduling and standard setting.
- Describe and identify the situations where learning effect can be incorporated.
- Calculate average hours/cost per unit of cumulative productions and incremental hours/cost per order using the learning curve.

16.1 Introduction

Learning is the process by which an individual acquires skill, knowledge and ability. When a new product or process is started, performance of worker is not at its best and learning phenomenon takes place. As the experience is gained, the performance of worker improves, time taken per unit reduces and thus his productivity goes up. This improvement in productivity of workers is due to learning effect. Cost predictions especially those relating to direct labour must allow for the effect of learning process. This technique is a mathematical technique. It is a graphical technique used widely to predict cost. Learning curve is a geometrical progression, which reveals that there is steadily decreasing cost for the accomplishment of a given repetitive operation, as the identical operation is increasingly repeated. The amount of decrease will be less and less with each successive unit produced. The slope of the decision curve is expressed as a percentage. The other names given to learning curve are Experience curve, Improvement curve and Progress curve. It is essentially a measure of the experience gained in production of an article by an organisation. As more units are produced, people involved in production become more efficient than before. Each additional unit takes less time to produce. The amount of improvement or experience gained is reflected in a decrease in man-hours or cost. The application of learning curve can be extended to commercial and industrial activities as well as defence production.

The learning effect exists during a worker’s start up or familiarization period on a particular job. After the limits of experimental learning are reached, productivity tends to stabilise and no
further improvement is possible. The rate at which learning occurs is influenced by many factors including the relative unfamiliarity of workers with the task, the relative novelty and uniqueness of the job, the complexity of the process, the impact of incentive plans, supervision, etc.

### 16.2 Distinctive Features of Learning Curve Theory in Manufacturing Environment

T.P. Wright of Curtiss—Wright, Buffalo, U.S.A. introduced the theory of learning curve. When the production quantity of a given item is doubled, the cost of that item decrease at a constant rate. Theory of learning curve has been formulated on the basis of this phenomenon. It is important to note that as the quantity produced doubles, the absolute amount of cost increase will be successively lessor but the rate of decrease will remain constant.

Features of learning curve can be summarized as below:

1. Better tooling methods are generated and applied;
2. More equipments are designed and used to increase the production;
3. Designed bugs are find and rectified;
4. Better design is achieve through design engineering for reducing material and labour cost.
5. Rejections and rework tend to reduce.

This results into: (a) less labour; (b) less material; (c) more units produced from the same equipments; (d) cost of fewer delays and less loss time. It allows increase of production and reduction of cost per unit.

### 16.3. The Learning Curve Ratio

In the initial stage of a new product or a new process, the learning effect pattern is so regular that the rate of decline established at the outset can be used to predict labour cost well in advance. The effect of experience on cost is summaries in the learning ratio or improvement ratio:

\[
\text{Learning Curve Ratio} = \frac{\text{Average labour cost of first } 2N \text{ units}}{\text{Average labour cost of first } N \text{ units}}
\]

If the average labour cost for the first 500 units of a product is ₹ 25 and the average labour cost of first 1,000 units is ₹ 20, the learning ratio will be determined as follow:

\[
\text{Learning curve ratio} = \frac{₹ 20}{₹ 25} \times 100 = 80\%
\]

This learning curve ratio of 80% means that every time output doubles, the average cost declines to 80% of the previous amount. Since the average cost per unit of 1,000 units is ₹ 20, the average cost of first 2,000 units is likely to be 80% of this, i.e., ₹ 16 per unit. The amount
of production improvement in the manufacturing of an article will determine the percentage of
the learning curve. The effect of learning can be presented clearly in a diagram drawn known
as learning curve. The units chosen in the progression must always have a ratio of two (Unit 2
to unit 1, unit 50 to unit 25, unit 300 to unit 150, etc.). An 80\% leaning curve is drawn using the
following data.

<table>
<thead>
<tr>
<th>Incremental quantity</th>
<th>Cumulative quantity</th>
<th>Workings of average time</th>
<th>Average time per unit</th>
<th>Cumulative time taken</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>(100 × 80)</td>
<td>80</td>
<td>160</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>(80 × 80)</td>
<td>64</td>
<td>256</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>(64 × 80)</td>
<td>51</td>
<td>408</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>(51 × 80)</td>
<td>41</td>
<td>656</td>
</tr>
</tbody>
</table>

Columns (2) and (4) have been used for drawing the learning curve. Last column is not used
in drawing the 80 per cent learning curve. This column shows how the cumulative time
consumption will increase with decrease in cumulative average per unit. Cumulative quantity is
Learning Curve Theory

16.4 Learning Curve Equation

Mathematicians have been able to express relationship in equations. The basic equation

\[ Y_x = KX^s \]  
*(it can be expressed as \( y = ax^{-b} \)) \( ...(1) \)

* even sometime it is expressed as \( y = ax^b \), though \( b \) always has a negative value

where,

\( X \) is the cumulative number of units or lots produced

\( Y \) is the cumulative average unit cost of those units \( X \) or lots.

\( K \) is the average cost of the first unit or lot

\( s \) is the improvement exponent or the learning coefficient or the index of learning which is calculated as follows:

\[ s = \frac{\log \text{ of learning ratio}}{\log \text{ of 2}} \]

Learning curve equation \( Y_x = KX^s \) becomes a linear equation when it is written in its logarithmic form:

\[ \log Y_x = \log K + s \log X \]  
\( ...(2) \)

Each of the above two equations defines cumulative average cost. Either of them can be converted to a formula for the total labour cost of all units produced up to a given point. Total cost under equation I can be found out by the following formula:

\[ \text{Total cost} = XY_x = KX^{s+1} \]  
\( ...(3) \)

16.5 Learning Curve Application

Knowledge of learning curve can be useful both in planning and control. Standard cost for new operations should be revised frequently to reflect the anticipated learning pattern. Its main uses are summarised below:

16.5.1 Helps to analyse CVP Relationship during familiarisation phase: Learning curve is useful to analyse cost-volume-profit relationship during familiarisation phase of product or process and thus it is very useful for cost estimates. Learning curve can be used as a tool for forecasting.
Illustration 1

XYZ Co., has observed that a 90% learning curve ratio applies to all labour related costs each time a new model enters production. It is anticipated that 320 units will be manufactured during 2012. Direct labour cost for the first lot of 10 units amounts to 1,000 hours at ₹8 per hour. Variable overhead cost is assigned to products at the rate of ₹2 per direct labour hour. You are required to determine:

(i) Total labour and labour-related costs to manufacture 320 units of output.
(ii) Average cost of (a) the first 40 units produced (b) the first 80 units, (c) the first 100 units.
(iii) Incremental cost of (a) units 41–80 and (b) units 101–200.

Solution

Table showing cost projections based on 90 per cent learning curve.

<table>
<thead>
<tr>
<th>Incremental quantity</th>
<th>Cumulative quantity</th>
<th>Average time per unit</th>
<th>Cumulative time taken</th>
<th>Incremental hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>100</td>
<td>1,000</td>
<td>—</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>90</td>
<td>1,800</td>
<td>800</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>81</td>
<td>3,240</td>
<td>1,400</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
<td>72.9</td>
<td>5,832</td>
<td>2,592</td>
</tr>
<tr>
<td>80</td>
<td>160</td>
<td>65.61</td>
<td>10,497.6</td>
<td>4,665.6</td>
</tr>
<tr>
<td>160</td>
<td>320</td>
<td>59.049</td>
<td>18,895.68</td>
<td>8,398.08</td>
</tr>
</tbody>
</table>

Following cost information can be derived from the data given in the above table:

(i) Total cost of 320 units = 18,895.68 × ₹10 = ₹1,88,956.80
(ii) Average cost of first 40 units = 81 × ₹10 = ₹810 per unit.
    Average cost of first 80 units = 72.9 × ₹10 or ₹729 per unit.
(iii) Incremental cost of units 41 – 80 = 2592 × ₹10 or ₹25,920.

The basic learning curve formula must be used to derive the average cost of the first 100 units and the incremental cost of units 101 – 200.

We know that $Y_x = KX^a$

The first production lot contained 10 units. $K = 1,000$ hrs. (time taken for the first lot) and $X = 10$ (the number of lots needed to produce 100 units).

Taking the logarithm of the above relation, we get

$$\log Y_{10} = \log 1000 + (-0.1518 \log 10)$$

$$= 3.0 - 0.1518$$

$$= 2.8482$$

$Y_{10} = 705$
Y_{10} means that average processing time for the first 10 lots of output was 705 hours per lot. Each lot includes 10 units of output. Therefore, average processing time for each of the first 100 units is 70.5 hours. The labour rate is ₹ 10 per hour. Therefore, the average cost of the first 100 units was ₹ 705 per unit. The incremental cost of units 101 – 200 corresponds to the incremental cost of lots 11 – 20. This is equal to the total cost of first 20 lots minus the total cost of the first 10 lots.

\[
\begin{align*}
Y_{10} &= 705 \\
Y_{10} &= (1000)(20 – 0.1518) = 634.5 \\
\text{Time for first 20 lots} &= 20(634.5) = 12,690 \text{ hours} \\
\text{Time of first 10 lots} &= 10(705) = 7,050 \text{ hours}
\end{align*}
\]

This means that incremental cost of lots 11 – 20 will be:

\[
(5,640 \text{ hrs. } \times ₹ 10) = ₹ 56,400
\]

**16.5.2 Helps in budgeting and profit planning:** Budget manager should select those costs which reflect learning effect and then he should be able to incorporate this effect in process of developing budgets or in the exercises relating to project planning.

**16.5.3 Helps in pricing:** The use of cost data adjusted for learning effect helps in development of advantageous pricing policy.

**16.5.4 Design makers:** It helps design engineers in making decisions based upon expected (predictable from past experience) rates of improvement.

**16.5.5 Helps in negotiations:** It is very useful to Government in negotiations about the contracts.

**16.5.6 Helps in setting standards:** The learning curve is quite helpful in setting standards in learning phase.

### 16.6 Application of Learning Curve

Learning curve can be applied in the following are:

1. **Though learning curve theory was first developed in aircraft industry, however, it can be applied to other manufacturing industry as well.**

2. *Learning curve can be applied even to non-production activities like marketing.*

3. *It can be very effective in labour oriented industry.*

4. *It can be effective with the job which is repetitive in nature particulars with same machinery and tools.*
16.7 Advanced Management Accounting

16.7 Financial Costs effected by Learning Curve

In general following costs are effected by learning curve:

1. Direct labour cost.
2. Variable overhead cost.

16.8 Important Consequences of learning curve in Management Accounting

In the following areas of management accounting effect of learning curve applies.

1. Variable overheads.
2. Standard costing.
3. Pricing decision.
4. Output capacity.
5. Direct labour budget.

16.9 Limitations of Learning Curve Theory

Following limitations of learning curve must be kept in view:

1. All activities of a firm are not subject to learning effect. Following types of activities are subject to learning effect:
   (a) Those, that have not been performed in this present operational mode.
   (b) Those which are being performed by new workmen, new employees or others not familiar with the particular activity. In contrast, activities being performed by experienced workmen, who are thoroughly familiar with those activities will not be subject to learning effect.
   (c) Those involving utilization of material not used by firm so far.

2. It is correct that learning effect does take place and average time taken is likely to reduce. But in practice it is highly unlikely that there will be a regular consistent rate of decrease, as exemplified earlier. Therefore any cost predictions based on covernational learning curves should be viewed with caution.

3. Considerable difficulty arises in obtaining valid data, that will form basis for computation of learning effect.

4. Even slight change in circumstances quickly renders the learning curve obsolete. While the regularity of conventional learning curves can be questioned, it would be wrong to ignore learning effect altogether in predicting future costs for decision purposes.
Illustration 2

A customer has asked your company to prepare a bid on supplying 800 units of a new product. Production will be in batches of 100 units. You estimate that costs for the first batch of 100 units will average ₹ 100 a unit. You also expect that a 90% learning curve will apply to the cumulative labour cost on this contract.

Required:

(a) Prepare an estimate of the labour costs of fulfilling this contract.

(b) Estimate the incremental labour cost of extending the production run to produce an additional 800 units.

(c) Estimate the incremental labour cost of extending the production run from 800 units to 900 units.

Solution

(a) Average cost decreases by 10 per cent every time when the cumulative production doubles.

Therefore,

Average cost of first 200 units = 0.9 × Avg. cost of 100 units
Average cost of first 400 units = 0.9 × Avg. cost of 200 units
Average cost of first 800 units = 0.9 × Avg. cost of 400 units

Combining these, we find that average cost of the first 800 units

= 0.9 × 0.9 × 0.9 × Rs.100 = ₹ 72.90

Total cost = 800 × ₹ 72.90 = ₹ 58,320

(b) Average cost of the first 1,600 units = 0.9 × ₹ 72.9 = ₹ 65.61

∴ Total cost of 1,600 units = 1,600 × ₹ 65.61 = ₹ 1,04,976

Additional cost of 2nd 800 units

= ₹ 1,04,976 – ₹ 58,320 = ₹ 46,656 or ₹ 58.32 per unit (46,656/800 units)

(c) Because this increase will not increase cumulative production to twice of some figure we already have, formula has to be used:

\[ Y = ax^b \]

where \( b = 0.0458 \div 0.301 = -0.15216 \)

\[ \log ax \text{ cost} = \log 10,000 - 0.15216 \log 9 \]

\[ = 4 - 0.1452 = 3.8548 \]

Average cost = ₹ 71.5833 per unit

Total cost = 900 × 71.5833 = ₹ 64,425

Incremental cost = ₹ 64,425 – ₹ 58,320 = ₹ 6,105 or ₹ 61.50 per unit.
Illustration 3
A company has 10 direct workers, who work for 25 days, a month of 8 hours per day. The estimated down time is 25% of the total available time. The company received an order for a new product. The first unit of the new product requires 40 direct labour hours to manufacture the product. The company expects 80% (index is – 0.322) learning curve for this type of work. The company uses standard absorption costing and the cost data are as under:

<table>
<thead>
<tr>
<th></th>
<th>₹ per unit/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct materials</td>
<td>60</td>
</tr>
<tr>
<td>Direct labour</td>
<td>6</td>
</tr>
<tr>
<td>Variable overheads</td>
<td>1</td>
</tr>
<tr>
<td>Fixed overheads</td>
<td>7,500</td>
</tr>
</tbody>
</table>

Required:
1. Calculate the cost per unit of the first order of 30 units.
2. If the company receives a repeat order for 20 units, what price will be quoted to yield a profit of 25% on selling price?

Solution

Working Notes:
1. Learning coefficient:
   \[ b = \frac{\log(80)}{\log(20)} = -0.3219 = -0.322 \] (Given in the problem)
2. Learning curve rate is expressed by
   \[ y = ax^{-b} \]
   Taking log of both sides, we have
3. Average labour time per unit for a batch of 30 units order
   For 30 units, \( y = (30)(40)(30)^{-0.3219} \)
   or \( \log y = \log 30 + \log 40 - 0.322(\log 30) \)
   \[ = 1.4771 + 1.6021 - 0.322(1.4771) \]
   \[ = 1.4771 + 1.6021 - 0.4757 = 2.6035 \]
   \( y \) (or total time for 30 units) \[ = \text{Antilog} 2.6032 = 401.4 \]
4. The first order is for 30 units. Second order for 20 units, then cumulative order will be for 50 units.
   Total time for 50 units = (50) (c) (50)-0.322
   \[ \log (\text{Total time of 50 units}) = \log 50 + .40 - 0.322(\log 50) \]
   \[ = 1.6990 + 1.6021 - 0.322(1.6990) \]
   \[ = 1.6990 + 1.6021 - 0.5471 = 2.754 \]
   Total time for 50 units = Antilog 2.754 = 567.5 hours
(v) Fixed overhead recovery rate per labour hour

10 men x 25 days x 8 hours = 2,000 hours
Less: Down time 25% = 500 hours
Total effective hours = 1,500 hours
Total fixed overhead for month = ₹ 7,500
Fixed overhead rate per labour hour = ₹ 7,500 / 1,500 = ₹ 5

(1) Computation of cost per unit of the first order of 30 units

- Direct material 360 x ₹ 60 = ₹ 1,800.00
- Direct labour 401.4 hrs x ₹ 6 = 2,408.40
- Variable overhead 401.4 hrs x ₹ 1 = 401.40
- Fixed overhead 401.4 hrs x ₹ 5 = 2,007.00
Total cost of 30 units = ₹ 6,616.80
Cost per unit = ₹ 6,616.80 / 30 = ₹ 220.56

(2) Cost per units, when a repeat order for 20 units is also placed

- Direct material (20 units x ₹ 60) = ₹ 1,200.00
- Direct labour (567.5 - 401.40) x ₹ 6 = 996.60
- Variable overhead (567.5 - 401.40) x ₹ 1 = 166.10
- Fixed overhead (567.5 - 401.40) x ₹ 5 = 830.50
Total cost of 50 additional units = ₹ 3,193.20
Cost per unit = ₹ 3,193.20 / 50 = ₹ 159.66

Price to be quoted to yield a profit of 25% on selling price
Profit required = 25% of selling price
Profit = ₹ 159.66 x 33 1/3% = ₹ 53.22
= ₹ 159.66 + 53.22 = ₹ 212.88

SUMMARY

- Learning Curve Effect applies only to direct labour costs and those variable overheads, which are direct function of labour hours of input. It does not apply to material costs, non-variable costs or items which vary with output.
- Incremental hours cannot be directly determined from the learning curve graph or formula, as the results are expressed in terms of cumulative average hours.