Risk and Return

CA Final Paper 2 Strategic Financial Management Chapter 7
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Learning Objectives

1. Discuss the objectives of portfolio Management - Risk and Return
2. Phases of Portfolio Management
3. Portfolio theories
4. Risk Analysis
5. Capital Asset Pricing Model (CAMP) in the valuation of securities
6. Arbitrage Pricing Theory (APT)
7. Sharpe Index Model
Introduction

The age-old wisdom about not putting “all your eggs in one basket” applies very much in the case of portfolios.

A portfolio is a combination of multiple securities.

Decisions to invest wealth in assets or securities under risk.

Extend the portfolio theory to derive a framework for valuing risky assets.

Portfolio Approaches

- Traditional
- Modern
Portfolio Investment Avenues

- Gold
- Silver
- Real Estate
  - Indira Vikas Patra
  - Post Office Deposits
  - Bank Deposits
  - NSC
- Shares
- Bonds
- Mutual Funds
- Debentures
- PF
Investment Parameters

– Return
– Risk
– Time Horizon
– Tax Considerations
– Liquidity
– Marketability
Risk-Return Trade off

Return

Risk

• Derivatives

• Shares
  • MFs Equity Fund

• Real Estate

• MFs Debt Funds
• Debentures

• Bonds, Bank Deposits

• NSC, Post-Office Deposit, PF
Portfolio Return: Traditional Approach

Analysis of constraints
(needs, liquidity, safety of principal, time horizon, tax consideration and temperament)

Determination of objectives
(current income, income growth, capital appreciation and preservation of capital)

Selection of portfolio

Bond & Common stock

Bond

Common stock

Assessment of Risk & Return

Diversification
Portfolio Risk:

Risk of individual assets is measured by their variance or standard deviation.

We can use variance or standard deviation to measure the risk of the portfolio of assets as well.

The risk of portfolio would be less than the risk of individual securities, and that the risk of a security should be judged by its contribution to the portfolio risk.
Elements of Risk

- Systematic Risk
  - Interest Rate Risk
  - Market Risk
  - Purchasing Power Risk
- Unsystematic Risk
  - Business Risk
  - Financial Risk
Diversification of Risk
Modern approach

Morkowitz model is

- Analysis of risk and return
- Inter-relationships through the statistical analysis for measuring risk

We can use the following equation to calculate the expected rate of return of individual asset:

\[ E(R_x) = (R_1 \times P_1) + (R_2 \times P_2) + \]
\[ (R_3 \times P_3) + \ldots + (R_n \times P_n) \]
Portfolio Investment Avenues

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Risk

- Derivatives
- Shares
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We can use the following equation to calculate the expected rate of return of individual asset:

\[
E(R_x) = (R_1 \times P_1) + (R_2 \times P_2) + (R_3 \times P_3) + \ldots + (R_n \times P_n)
\]
Expected Rate of Return: Example

<table>
<thead>
<tr>
<th>Possible returns (in %) $X_i$</th>
<th>Probability of occurrence $p_i(X_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.20</td>
</tr>
<tr>
<td>30</td>
<td>0.20</td>
</tr>
<tr>
<td>50</td>
<td>0.40</td>
</tr>
<tr>
<td>60</td>
<td>0.10</td>
</tr>
<tr>
<td>70</td>
<td>0.10</td>
</tr>
</tbody>
</table>

*Expected* Rate of Return Based on Probabilities =

$$
\overline{X} = \sum_{i=1}^{n} X_i \ p_i(X_i)
$$
Calculation of Expected Return

<table>
<thead>
<tr>
<th>Possible returns $X_i$</th>
<th>Probability $p_i(X_j)$</th>
<th>$X_i p_i(X_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.20</td>
<td>4.00</td>
</tr>
<tr>
<td>30</td>
<td>0.20</td>
<td>6.00</td>
</tr>
<tr>
<td>40</td>
<td>0.40</td>
<td>16.00</td>
</tr>
<tr>
<td>50</td>
<td>0.10</td>
<td>5.00</td>
</tr>
<tr>
<td>60</td>
<td>0.10</td>
<td>6.00</td>
</tr>
<tr>
<td>$\sum_{i=1}^{n} X_i p_i(X_i)$</td>
<td></td>
<td>37.00</td>
</tr>
</tbody>
</table>

Hence the expected return is 37 per cent
## Risk Calculation - Security Variance

<table>
<thead>
<tr>
<th>Possible returns $X_i$</th>
<th>Probability $p_i(X_j)$</th>
<th>Deviation $(x_i - \bar{x})$</th>
<th>Deviation squared $(x_i - \bar{x})^2$</th>
<th>Product $(x_i - \bar{x})^2 \cdot p_i(x_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.20</td>
<td>-17.00</td>
<td>289.00</td>
<td>57.80</td>
</tr>
<tr>
<td>30</td>
<td>0.20</td>
<td>-7.00</td>
<td>49.00</td>
<td>9.80</td>
</tr>
<tr>
<td>40</td>
<td>0.40</td>
<td>3.00</td>
<td>9.00</td>
<td>3.60</td>
</tr>
<tr>
<td>50</td>
<td>0.10</td>
<td>13.00</td>
<td>169.00</td>
<td>16.90</td>
</tr>
<tr>
<td>60</td>
<td>0.10</td>
<td>23.00</td>
<td>529.00</td>
<td>52.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Variance</strong></td>
<td><strong>141.00</strong></td>
</tr>
</tbody>
</table>

Security variance = $[p_1 \times (R_1 - E(R_1))^2 + p_2 \times (R_2 - E(R_2))^2 + \ldots + p_n \times (R_n - E(R_n))^2]$
The portfolio variance or standard deviation depends on the co-movement of returns on two assets.

Covariance of returns on two assets measures their co-movement.

Correlation is the measure of the linear relationship between two variables (say, returns of two securities, X and Y in our case).

\[
\text{Covariance } XY = \text{Standard deviation } X \times \text{Standard deviation } Y \times \text{Correlation } XY
\]
Variance and Standard Deviation of a Two-Asset Portfolio

The variance of a two-asset portfolio is not the weighted average of the variances of assets since they co-vary as well.

The variance of two-security portfolio is given by the following equation:

\[ \sigma_p^2 = \sigma_x^2 w_x^2 + \sigma_y^2 w_y^2 + 2 w_x w_y \text{Cov}_{xy} \]

\[ = \sigma_x^2 w_x^2 + \sigma_y^2 w_y^2 + 2 w_x w_y (\sigma_x \sigma_y \text{Cor}_{xy}) \]
Covariance

Measures the co movement of securities.

3 steps to calculate Covariance.
1. Determine the expected returns on assets.
2. Determining the deviation of possible returns from the expected return for each asset.
3. Determining the sum of the product of each deviation of returns of two assets and respective probability.

\[
\text{Covariance between two sets of returns } A_1, \text{ and } A_2 \text{ is given by:}
\]

\[
\text{Cov} (A_1, A_2) = P_1 (A_1 - \overline{A}) (A_2 - \overline{A}) + P_2 (A_1 - \overline{A}) (A_2 - \overline{A})
\]

Corelation Coefficient \( \rho_{12} = \frac{\text{Cov}(A_1, A_2)}{\sigma_1 \sigma_2} \)
Possibilities of Covariance

• The relationship between the returns of securities X and Y have following possibilities:

  • Positive covariance : Implies positive relation between the two returns.
  • Negative covariance : Implies negative relation between the two returns.
  • Zero covariance : Implies no relation between the two returns.
The correlation of the two securities $X$ and $Y$ is as follows:

$$\text{Cor}_{xy} = \frac{-33.0}{5.80 \cdot 7.63} = \frac{-33.0}{44.25} = -0.746$$

Securities $X$ and $Y$ are negatively correlated. The correlation coefficient of $-0.746$ indicates a high negative relationship.
Correlation
It measures linear relationship between two variables

The value of correlation, called the correlation coefficient, could be positive, negative or zero.

The correlation coefficient always ranges between –1.0 and +1.0.

A correlation coefficient of +1.0 implies a perfectly positive correlation while a correlation coefficient of –1.0 indicates a perfectly negative correlation.
Positive Correlation (Perfect +1)

When correlation is +1.0, the portfolio risk (standard deviation) is simply given by the following formula:

\[ \sigma_p = \sqrt{\sigma_x^2 w_x^2 + \sigma_y^2 w_y^2 + 2w_x w_y \sigma_x \sigma_y} \]

\[ = \sigma_x w_x + \sigma_y w_y \]
There is no advantage of diversification when the returns of securities have perfect positive correlation.
When correlation is $-1.0$, the portfolio risk (standard deviation) is simply given by the following formula:

$$
\sigma_p = \sqrt{\sigma_x^2 w_x^2 + \sigma_y^2 w_y^2 - 2w_x w_y \sigma_x \sigma_y} = \text{ABS} \left[ \sigma_x w_x - \sigma_y w_y \right]
$$
Perfect Negative Correlation (-1)

• Portfolio risk declines & portfolio return increases.

• It results in risk-less portfolio.

• The correlation is -1.0.

\[ W_x = \frac{\sigma_y}{\sigma_x + \sigma_y} \]
Perfect Negative Correlation (-1)
Zero Correlation

- It indicates that the returns are independent of each other.

- No possibility of achieving riskless portfolio and standard deviation can not be reduce to zero.
Limits to diversification
Portfolio Return: Modern approach

\[ E(R_p) = w_1 E(R_1) + w_2 E(R_2) \]

where:
\[ E(R_p) \] = expected return on Portfolio P
\[ w_i \] = proportion ("weight") of the portfolio allocated to Asset i
\[ E(R_i) \] = expected return on Asset i

The weights (\( w_1 \) and \( w_2 \)) must sum to 100% for a two-asset portfolio.

The variance of a two-asset portfolio equals:

\[ \sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}_{1,2} \]

where:
\[ \sigma_P^2 \] = variance of the returns for Portfolio P
\[ \sigma_1^2 \] = variance of the returns for Asset 1
\[ \sigma_2^2 \] = variance of the returns for Asset 2
\[ w_i \] = proportion (weight) of the portfolio allocated to Asset i
\[ \text{Cov}_{1,2} \] = covariance between the returns of the two assets
Portfolio Return: Modern approach

Expected return & standard deviation for two asset portfolio

Using the information, calculate the expected return and standard deviation of the two asset portfolio

<table>
<thead>
<tr>
<th>Characteristics of the two asset portfolio</th>
<th>Marico</th>
<th>HUL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount investment (IN)</td>
<td>40000</td>
<td>60000</td>
</tr>
<tr>
<td>Expected return</td>
<td>11%</td>
<td>25%</td>
</tr>
<tr>
<td>Standard deviation'</td>
<td>15%</td>
<td>20%</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>
Portfolio Return: Modern approach

\[ E(R_p) = w_c E(R_c) + w_s E(R_s) \]

\[ E(R_p) = (0.40)(0.11) + (0.60)(0.25) = 0.1940 = 19.40\% \]

Then, we calculate the variance of the portfolio:

\[ \sigma_p^2 = w_c^2 \sigma_c^2 + w_s^2 \sigma_s^2 + 2w_c w_s \rho_{cs} \sigma_c \sigma_s \]

\[ = (0.40)^2 (0.15)^2 + (0.60)^2 (0.20)^2 + 2(0.40)(0.60)(0.30)(0.15)(0.20) \]

\[ = 0.02232 \]

And, finally, the standard deviation of the portfolio:

\[ \sigma_p = \sqrt{\sigma_p^2} = \sqrt{0.02232} = 0.1494 = 14.94\% \]
Consider the following information on two stocks, A and B:

<table>
<thead>
<tr>
<th>Year</th>
<th>Return on A (%)</th>
<th>Return on B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>2007</td>
<td>16</td>
<td>18</td>
</tr>
</tbody>
</table>

You are required to determine:

(i) The expected return on a portfolio containing A and B in the proportion of 40% and 60% respectively.

(ii) The Standard Deviation of return from each of the two stocks.

(iii) The covariance of returns from the two stocks.

(iv) Correlation coefficient between the returns of the two stocks.

(v) The risk of a portfolio containing A and B in the proportion of 40% and 60%.
Answer

(i) Expected return of the portfolio A and B
E (A) = (10 + 16) / 2 = 13%
E (B) = (12 + 18) / 2 = 15%
Rp = \sum_{i=1}^{N} X_i R_i = 0.4(13) + 0.6(15) = 14.2%

(ii) Stock A:
Variance = 0.5 (10 − 13)^2 + 0.5 (16 − 13)^2 = 9
Standard deviation = \sqrt{9} = 3%

Stock B:
Variance = 0.5 (12 − 15)^2 + 0.5 (18 − 15)^2 = 9
Standard deviation = 3%

(iii) Covariance of stocks A and B
Cov_{AB} = 0.5 (10 − 13) (12 − 15) + 0.5 (16 − 13) (18 − 15) = 9

(iv) Correlation of coefficient
r_{AB} = \frac{Cov_{AB}}{\sigma_A \sigma_B} = \frac{9}{3 \times 3} = 1

(v) Portfolio Risk
\sigma_p = \sqrt{X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B (\sigma_A \sigma_B \sigma_{AB})}
= \sqrt{(0.4)^2 (3)^2 + (0.6)^2 (3)^2 + 2(0.4)(0.6)(3)(3)(1)}
= \sqrt{1.44 + 3.24 + 4.32} = 3%
Mean-variance Criterion

- Portfolio opportunity set represents all possible combinations of risk and returns.
- Inefficient portfolios - have lower return and higher risk.
- Efficient portfolio has highest returns for a given level of risk.
- Efficient frontier is created by efficient portfolio.
- Inefficient portfolio lies outside the efficient frontier.
The efficient frontier is formed by the set of efficient portfolios. Efficient portfolio has the highest expected returns for a given level of risk. All other portfolios, which lie outside the efficient frontier, are inefficient portfolios.
Selection of Portfolios
Optimal Investment Under Markowitz Model
Markowitz Efficient Frontier
Combination of Risk-Free Asset and Risky Asset

Risk-return relationship for portfolio of risky and risk-free securities
Draw lines from the risk-free rate 7.5% capital allocation line.

Portfolio $M$ is the optimum risky portfolio.
LENDING AND BORROWING AT RISK FREE RATE

CAL IS A COMBINATION OF RISK FREE AND RISKY ASSETS
CAPITAL MARKET LINE
Separation Theory

Two steps for the combination of risk free and risky portfolio.

1. Determine optimum risky portfolio

2. Investors decision between Risk free & Risky portfolio.
Lending & Borrowing
RISK-FREE ASSET & RISKY ASSET

When risk-free and a risky asset are combined, the portfolio return is:

\[ E(R_p) = wE(R_j) + (1 - w)R_f \]

Where, \( R_f \) = risk-free security

\( R_j \) = risky security
Slope of CML

\[
\text{Slope of CML} = \frac{E(R_m) - R_f}{\sigma_m}
\]

EXPECTED PORTFOLIO RATE OF RETURN

\[
\text{CML}: \quad E(r) = r_f + \sigma \frac{E(r_M) - r_f}{\sigma_M}
\]
Assume that an investor has an opportunity to invest in a risk-free security $R$ of which he has an expected return of 7 per cent and market portfolio $P$ with an expected return of 15 per cent and a standard deviation of 6 per cent.

If the Investor Expected return on 12 %. What is the portfolio risk and What percentage he should invest risk free and risky securities?
Answer

- E(R) = \omega E(R_p) + (1-\omega)R_r
- 0.12 = \omega_p * 0.15 + (1-\omega_p)0.07
- 0.12 = \omega_p 0.15 + 0.07 - \omega_p 0.07
- \omega_p = 0.05/0.08 = 0.625
- 62.5%
- \omega_f = 1 - 0.625 = 0.375
- Since the risk-free security has a zero standard deviation and covariance between the risk-free security and risky security is zero, the portfolio risk is simply given as the product of the standard deviation of the risky security and its weight. Thus
  - \sigma_p = \omega \sigma_p
  - \sigma_p = 0.625 * 0.06 = 0.0375 or 3.75%
Expected portfolio rate of return

\[
\text{CML: } E(r) = r_f + \sigma \frac{E(r_M) - r_f}{\sigma_M}.
\]

\[
= 0.07 + \left( \frac{0.15 - 0.07}{0.06} \right) \times 0.0375
\]

\[
= 0.07 + \frac{0.08}{0.06} \times 0.0375
\]

\[
= 0.12
\]
Capital Asset Pricing Model (CAPM)

Determining the required rate of return on an asset.

Relationship Between Return & Risk

Compare Between the Expected Return & Required Return

SML

Explain the Relationship between an asset’s risk and its required rate of return.
Assumptions of CAPM

- Efficient Market
- Rational Investment Goals
- Homogeneous expectations
- Risk-free rate for Lending & Borrowing
Capital Asset Pricing Model (CAPM)

\[ \beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} \]

\[ \bar{r}_a = r_f + \beta_a (\bar{r}_m - r_f) \]

Where:
- \( r_f \) = Risk free rate
- \( \beta_a \) = Beta of the security
- \( \bar{r}_m \) = Expected market return
Security market line with normalize systematic risk (\(\beta\))
Undervalued & Over Valued Stocks

(a) When \( \text{CAPM} < \text{Expected Return} \) – Buy: This is due to the stock being undervalued i.e. the stock gives more return than what it should give.

(b) When \( \text{CAPM} > \text{Expected Return} \) – Sell: This is due to the stock being overvalued i.e. the stock gives less return than what it should give.

(c) When \( \text{CAPM} = \text{Expected Return} \) – Hold: This is due to the stock being correctly valued i.e. the stock gives same return than what it should give.

Actual Market Price < CAPM, stock is undervalued
Actual market Price > CAPM, stock is overvalued
Actual market Price = CAPM, stock is correctly valued.
### Undervalued & Over Valued Stocks

<table>
<thead>
<tr>
<th>Stock</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Return (%)</td>
<td>18</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>Beta Factor</td>
<td>1.7</td>
<td>0.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>

If the risk free rate is 9% and the expected rate of return on the market portfolio is 14% which of the above stocks are over, under or correctly valued in the market? What shall be the strategy?
Solution

Required Rate of Return is given by

\[ R_j = R_f + \beta (R_m - R_f) \]

For Stock A, \[ R_j = 9 + 1.7 \times (14 - 9) = 17.50\% \]

Stock B, \[ R_j = 9 + 0.6 \times (14 - 9) = 12.00\% \]

Stock C, \[ R_j = 9 + 1.2 \times (14 - 9) = 15.00\% \]

<table>
<thead>
<tr>
<th>Required Return %</th>
<th>Expected Return %</th>
<th>Valuation</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.50%</td>
<td>18.00%</td>
<td>Under Valued</td>
<td>Buy</td>
</tr>
<tr>
<td>12.00%</td>
<td>11.00%</td>
<td>Over Valued</td>
<td>Sell</td>
</tr>
<tr>
<td>15.00%</td>
<td>15.00%</td>
<td>Correctly Valued</td>
<td>Hold</td>
</tr>
</tbody>
</table>
The Arbitrage Pricing Theory (Apt)

Arbitrage Means buy low & Sell high

Mispriced assets means that the current price is different from the predicted price.

APT states that investors go for arbitrage whenever they find differences in return of assets with similar risk.
Concept of Risk under APT

APT assumes that market risk can be caused by economic factors such as changes in gross domestic product, inflation, and the structure of interest rates and these factors could affects the firms differently. Therefore, under APT the sensitivity of the asset’s return to each factor is estimated. For each firm, there will be as many betas as the number of factors.

\[
E(R_j) = R_f + (\beta_1 F_1 + \beta_2 F_2 + \beta_3 F_3 + \cdots + \beta_n F_n) + \mu R_s
\]
Steps in Calculating Expected Return under APT

1. searching for the factors that affect the asset’s return
2. estimation of risk premium for each factor
3. estimation of factor beta
Factors

- Industrial production
- Changes in default premium
- Changes in the structure of interest rates
- Inflation rate
- Changes in the real rate of return
Factor beta

• The beta of the factor is the sensitivity of the asset’s return to the changes in the factor.

• One can use regression approach to calculate the factor beta.
Arbitrage Pricing Theory Model (APT)

According to CAPM, \( E(R_i) = R_f + \lambda \beta_i \)

Where, \( \lambda \) is the average risk premium \([E(R_m) - R_f]\)

In APT, \( E(R_i) = R_f + \lambda_1 \beta_{i1} + \lambda_2 \beta_{i2} + \lambda_3 \beta_{i3} + \lambda_4 \beta_{i4} \)

Where, \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) are average risk premium for each of the four factors in the model and \( \beta_{i1}, \beta_{i2}, \beta_{i3}, \beta_{i4} \) are measures of sensitivity of the particular security \( i \) to each of the four factors.
An investor is considering to make an investment in the share of RIL. The following are the attributes of five economic forces that influence the return of RIL’s share:

<table>
<thead>
<tr>
<th>Factor Actual</th>
<th>beta</th>
<th>Risk Premium value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNP</td>
<td>2.00</td>
<td>2.00%</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.00</td>
<td>2.00%</td>
</tr>
<tr>
<td>Crude oil rate</td>
<td>1.50</td>
<td>1.00%</td>
</tr>
<tr>
<td>Stock market index</td>
<td>2.50</td>
<td>2.00%</td>
</tr>
<tr>
<td>Industrial Growth</td>
<td>2.00</td>
<td>1.00%</td>
</tr>
</tbody>
</table>

The risk-free (anticipated) rate of return on the RIL’s share is 9 per cent. How much is the total return on the share? The total return will consist of anticipated (risk-free) return and unanticipated return:

\[
E(R) = 9 + [(2.00)2.00 + (2.00)1.00 + (1.00)1.50 + (2.00)2.50 + (1.00)2.00] = 9 + 14.5 = 23.5\%
\]
Single Index Model

Stock prices are related to the market index.

Sensex increases, stock prices also tend to increase and vice-versa.

Co-movement between stocks is due to change or movement in the market index.
Single Index Model

\[ R_i = \alpha_i + \beta_i R_m + \epsilon_i \]

Where,

- \( R_i \) = expected return on security \( i \)
- \( \alpha_i \) = intercept of the straight line or alpha co-efficient
- \( \beta_i \) = slope of straight line or beta co-efficient
- \( R_m \) = the rate of return on market index
- \( \epsilon_i \) = error term.
Characteristics Line
Risk

Total risk of an individual security is the variance (or standard deviation) of its return. It consists of two parts:

\[
\text{Total risk of a security} = \text{Systematic risk} + \text{Unsystematic risk}
\]

Systematic risk is attributable to macroeconomic factors.

Above equation can also be written as:

\[
\text{Total risk} = \text{variance attributable to macroeconomic factors} + (\text{residual}) \text{ variance attributable to firm-specific factors}
\]

Total risk is not relevant for an investor who holds a diversified portfolio.
Variance of Security’s Return

The variance of security’s return:
\[ \sigma^2 = \beta^2 \sigma^2_m + \sigma^2_{ei} \]

The covariance of returns between securities i and j is:
\[ \sigma_{ij} = \beta_i \beta_j \sigma^2_m \]

Systematic risk = \( \beta^2_i \times \) variance of market index
\[ \sigma^2_m \]

Unsystematic risk = Total variance - Systematic risk.
\[ \epsilon^2_i = \sigma^2_i - \text{Systematic risk.} \]

Thus, the total risk = Systematic risk + Unsystematic risk.
\[ = \beta^2_i \sigma^2_m + \epsilon^2_i. \]
## Question

The rates of return on the security of company X and market Portfolio for 10 periods are given below:

<table>
<thead>
<tr>
<th>Period</th>
<th>Return of Security X (%)</th>
<th>Return on Market Portfolio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>-5</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>-6</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>-7</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>11</td>
</tr>
</tbody>
</table>

(i) What is the beta of Security X?

(ii) What is the characteristic line for Security X?
Answer

<table>
<thead>
<tr>
<th>Period</th>
<th>( R_X )</th>
<th>( R_M )</th>
<th>( R_X - \bar{R}_X )</th>
<th>( R_M - \bar{R}_M )</th>
<th>((R_X - \bar{R}_X)(R_M - \bar{R}_M))</th>
<th>((R_M - \bar{R}_M)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>22</td>
<td>5</td>
<td>10</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>20</td>
<td>7</td>
<td>8</td>
<td>56</td>
<td>64</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>18</td>
<td>10</td>
<td>6</td>
<td>60</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
<td>16</td>
<td>6</td>
<td>4</td>
<td>24</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>20</td>
<td>3</td>
<td>8</td>
<td>24</td>
<td>64</td>
</tr>
<tr>
<td>6</td>
<td>-5</td>
<td>8</td>
<td>-20</td>
<td>-4</td>
<td>80</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>-6</td>
<td>2</td>
<td>-18</td>
<td>-36</td>
<td>324</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
<td>5</td>
<td>4</td>
<td>-7</td>
<td>-28</td>
<td>49</td>
</tr>
<tr>
<td>9</td>
<td>-7</td>
<td>6</td>
<td>-22</td>
<td>-6</td>
<td>132</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>11</td>
<td>5</td>
<td>-1</td>
<td>-5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>120</td>
<td></td>
<td></td>
<td>(\sum (R_X - \bar{R}_X)(R_M - \bar{R}_M))</td>
<td>(\sum (R_M - \bar{R}_M)^2)</td>
</tr>
</tbody>
</table>

\( \bar{R}_X = 15 \) \( \bar{R}_M = 12 \)
\[
\sigma^2_M = \frac{\sum (R_M - \bar{R}_M)^2}{n} = \frac{706}{10} = 70.60
\]

\[
\text{Cov}_{xM} = \frac{\sum (R_X - \bar{R}_X)(R_M - \bar{R}_M)}{n} = \frac{357}{10} = 35.70
\]

\[
\text{Beta}_X = \frac{\text{Cov}_{xM}}{\sigma^2_M} = \frac{35.70}{70.60} = 0.505
\]

(ii) \[\bar{R}_X = 15 \quad \bar{R}_M = 12\]

\[y = \alpha + \beta x\]

\[15 = \alpha + 0.505 \times 12\]

\[
\text{Alpha (}\alpha\text{)} = 15 - (0.505 \times 12)
\]

\[= 8.94\%\]

Characteristic line for security \(X = \alpha + \beta \times R_M\)

Where, \(R_M = \) Expected return on Market Index

\[\therefore \text{Characteristic line for security } X = 8.94 + 0.505 \times R_M\]
Illustration 8: The following details are given for X and Y companies’ stocks and the Bombay Sensex for a period of one year. Calculate the systematic and unsystematic risk for the companies’ stocks. If equal amount of money is allocated for the stocks what would be the portfolio risk?

<table>
<thead>
<tr>
<th></th>
<th>X Stock</th>
<th>Y Stock</th>
<th>Sensex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average return</td>
<td>0.15</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>Variance of return</td>
<td>6.30</td>
<td>5.86</td>
<td>2.25</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.71</td>
<td>0.685</td>
<td></td>
</tr>
<tr>
<td>Correlation Co-efficient</td>
<td>0.424</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co-efficient of determination ((r^2))</td>
<td>0.18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The co-efficient of determination ($r^2$) gives the percentage of the variation in the security’s return that is explained by the variation of the market index return. In the X company stock return, 18 per cent of variation is explained by the variation of the index and 82 per cent is not explained by the index.

According to Sharpe, the variance explained by the index is the systematic risk. The unexplained variance or the residual variance is the unsystematic risk.

**Company X:**

Systematic risk

$$\beta_i^2 \times \text{Variance of market index}$$

$$= (0.71)^2 \times 2.25$$

$$= 1.134$$

Unsystematic risk ($\xi_1^2$)

$$= \text{Total variance of security return - systematic risk}$$

$$= 6.3 - 1.134$$

$$= 5.166$$

or
Answer Cont...

\[ = \text{Variance of Security Return } (1-r^2) \]
\[ = 6.3 \times (1-0.18) \]
\[ = 6.3 \times 0.82 = 5.166 \]

Total risk
\[ = \beta_i^2 \times \sigma_m^2 + \varepsilon_i^2 \]
\[ = 1.134 + 5.166 = 6.3 \]

Company Y:

Systematic risk
\[ = \beta_i^2 \times \sigma_m^2 \]
\[ = (0.685)^2 \times 2.25 = 1.056 \]

Unsystematic risk
\[ = \text{Total variance of the security return - systematic risk.} \]
\[ = 5.86-1.056 = 4.804 \]

\[ \sigma_p^2 = \left[ \left( \sum_{i=1}^{N} X_i \beta_i \right)^2 \sigma_m^2 \right] + \left[ \left( \sum_{i=1}^{N} X_i^2 \varepsilon_i^2 \right) \right] \]
\[ = [(0.5 \times 0.71 + 0.5 \times 0.685)^2 \times 2.25] + [(0.5)^2(5.166)+(0.5)^2(4.804)] \]
\[ = [(0.355 + 0.3425)^2 \times 2.25] + [(1.292 + 1.201)] \]
\[ = 0.4865 + 2.493 = 2.9795 \]
ACTIVE PORTFOLIO STRATEGY

Market Timing

Sector Rotation

Security Selection

Use of Specialised Investment Concept
PASSIVE PORTFOLIO STRATEGY

Well diversified portfolio at a predetermined level of risk.

Index funds are passively managed funds.
SELECTION OF SECURITIES

- Selection of Bonds
- Selection of Stock
SELECTION OF BONDS

Yield to Maturity

Risk of Default

Tax Shield:

Liquidity
LEVEL OF MARKET EFFICIENCY

• Weak form efficiency

• Semi – Strong efficiency

• Strong from efficiency
## SELECTION OF STOCK

Technical Analysis
Fundamental Analysis
Random Selection Analysis

---

### Levels Of Market Efficiency And Approach To Security Selection

<table>
<thead>
<tr>
<th>Levels of Efficiency</th>
<th>Approach</th>
<th>Technical Analysis</th>
<th>Fundaments Analysis</th>
<th>Random Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Inefficiency</td>
<td></td>
<td>Best</td>
<td>Poor</td>
<td>Poor</td>
</tr>
<tr>
<td>2) Weak form efficiency</td>
<td></td>
<td>Poor</td>
<td>Best</td>
<td>Poor</td>
</tr>
<tr>
<td>3) Semi-strong efficiency</td>
<td></td>
<td>Poor</td>
<td>Good</td>
<td>Fair</td>
</tr>
<tr>
<td>4) Strong Form efficiency</td>
<td></td>
<td>Poor</td>
<td>Fair</td>
<td>Best</td>
</tr>
</tbody>
</table>
RANDOM WALK THEORY

• Prices of shares in stock market can never be predicted.
• The price trends are not the result of any underlying factors, but that they represent a statistical expression of past data.
• No connection can be established between two successive peaks (high price of stocks) and troughs (low price of stocks).
Thank you